Section 1.7 Solving Quadratic Equations

An equation that can be written in the form $ax^2 + bx + c = 0$ is a quadratic equation. In this section we will learn strategies for solving any quadratic equation.

Solve: $x^2 = 16$

$$x = \pm 4$$

Solve: $(2x + 3)^2 = 5$

$2x+3 = \sqrt{5}$ or $2x+3 = -\sqrt{5}$

$x = \dfrac{\sqrt{5}-3}{2}$ or $x = \dfrac{-\sqrt{5}-3}{2}$

Extracting Square Roots.
When possible, isolate a perfect square term by setting it equal to a constant and then simplify by setting the quantity that is squared equal to the positive and negative square root of the other side.

Solve: $2x^2 = x + 6$

$$2x^2 - x - 6 = 0$$

$$(2x + 3)(x - 2) = 0$$

$$x = -\frac{3}{2} \quad \text{or} \quad x = 2$$

Solve: $(x + 4)^2 = 13x + 10$

$$x^2 + 8x + 16 = 13x + 10$$

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

$$x = 3 \quad \text{or} \quad x = 2$$

The Zero Factor Property and Factoring.
When possible, set the quadratic equal to zero and then factor the quadratic.
Set each factor equal to zero and solve.

Solve: $2x^2 + 3x = 5$

$$2x^2 + 3x - 5 = 0$$

$$(2x + 5)(x - 1) = 0$$

$$x = -\frac{5}{2} \quad \text{or} \quad x = 1$$

Solve: $(x - 1)(x + 2) = 4x(x + 2)$

$$x^2 - x + 2x - 2 = 4x^2 + 8x$$

$$x^2 + x - 2 = 4x^2 + 8x$$

$$-3x^2 - 7x - 2 = 0$$

$$3x^2 + 7x + 2 = 0$$

$$(3x + 1)(x + 2)$$

Quadratic Formula.
Write the quadratic in general form $ax^2 + bx + c = 0$ Then plug a, b, and c into the formula below and simplify.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$